Let V be the intersection of AE and BC. Then

$$\sin^2\frac{A}{2}=\frac{IH}{AI}\cdot\frac{EG}{AE},\qquad \cos^2\frac{A}{2}=\frac{AH}{AI}\cdot\frac{AG}{AE}.$$

Since BI and CE (not drawn in the figure) are angle-bisectors,

$$\frac{AI}{IV}=\frac{AB}{BV},\qquad \frac{AE}{VE}=\frac{AC}{VC}.$$

But

$$IV\cdot VE=BV\cdot VC.$$

Hence

$$AI\cdot AE=AB\cdot AC=bc.$$

Therefore

$$\sin^2\frac{A}{2}=\frac{(s-b)(s-c)}{bc},\qquad \cos^2\frac{A}{2}=\frac{s(s-a)}{bc}.$$

III. The Use of the Vector in Analytical Geometry.

By Vincent C. Poor, University of Michigan.

In a first course in analytical geometry it is necessary to adhere rather closely to the text, whatever the text. This is true not because of the nature of the subject matter but because of the mathematical immaturity of the students electing the subject. Important innovations thus furnish one excuse for another textbook.

In many of the textbooks on analytical geometry the directed line is not mentioned at all. This is deplorable from the point of view of the physicist, for the geometric interpretation of many physical quantities leads to simplicity in expression and clearness in comprehension. Aside from this need the subject of analytical geometry may, in my opinion, be much more easily and directly presented if a more extended use of the vector be made.

The ground work for this is to be found in some of our textbooks, *e.g.*, Woods and Bailey, *A Course in Mathematics*, Vol. I; Ziwet and Hopkins, *Analytic Geometry*. In their study of directed lines we find the equivalents of the following theorems:

Theorem I. *The projection of a line segment on another line is equal to the length of the line segment into the cosine of their included angle.*

Theorem II. *The projection of a broken line on another line is equal to the projection of the join of its end points.*

In a number of the books the fundamental theorem of the geometry of segments is deduced, namely

Theorem III. *Given three points, O, P, Q, on a directed line, then*

$$PQ=OQ-OP,$$

in magnitude and sense.

If O is taken as the origin and the coördinates x_1 and x_2 be assigned to the

points P and Q respectively we can write

$$PQ = x_2 - x_1$$

in magnitude and sense.

Although these three theorems are given in some of our textbooks, very little use is made of them in the further development of the subject. Students may well wonder why they have a place in the book at all. And yet if they were used the usual theorems of analytical geometry expressed in rectangular coördinates could be easily and rigorously proved, the proofs being general and not "piecemeal" as is the case in possibly all of our texts. A few illustrations will serve to show this.

After defining the rectangular coördinate system, we may consider a directed line, a vector, anywhere in the plane. Call its initial point $P(x_1, y_1)$ and its terminal point $P_2(x_2, y_2)$. Let d and θ be its length and vectorial angle respectively. Then the projections P_1' and P_2' of P_1 and P_2 on the axis OX will be given by the coördinates x_1 and x_2 respectively, and by Theorem III

$$P_1'P_2' = x_2 - x_1$$

furnishes the projection of the vector d on the axis OX in magnitude and sense. According to Theorem I this projection is given by $d \cos \theta$, so that withc onsiderable ease one arrives at the fundamental equations of projection:

$$\begin{aligned} x_2 - x_1 &= d \cos \theta, \\ y_2 - y_1 &= d \sin \theta. \end{aligned} \tag{1}$$

Squaring and adding we obtain the distance formula. Dividing the second of (1) by the first, we obtain the slope of the vector. If $P_3(x_3, y_3)$ divides d into segments such that

$$P_1P_3/P_1P_2 = k$$

then calling the segment P_1P_3, d_1, we may write:

$$x_3 - x_1 = d_1 \cos \theta, \qquad x_2 - x_1 = d \cos \theta.$$

Dividing, one has

$$(x_3 - x_1)/(x_2 - x_1) = k,$$

or

$$x_3 = x_1 + k(x_2 - x_1).$$

The equation of a line having a given direction and passing through a given point may be written at once from the formula furnishing the slope of the line by considering (x_2, y_2) the coördinates of any point on the line. The "normal form" for the straight line follows in a very brief way from Theorems I and II. The coördinates of any point (x, y) on the line may be interpreted as segments of a broken line, and its projection $x \cos \beta + y \sin \beta$, on p, the perpendicular from the origin, with vectorial angle β, is the projection of ρ, the join of its end points, or p itself. From this, of course, the distance from a point to a line follows. If

the equation of the line through the two points (x_2, y_2) and (x_3, y_3) be written in determinantal form, the distance h from the point (x_1, y_1) to the line is

$$h = \frac{\begin{vmatrix} x_1, & y_1, & 1 \\ x_2, & y_2, & 1 \\ x_3, & y_3, & 1 \end{vmatrix}}{\sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}}.$$

Since the denominator is the base of the triangle determined by the three points, the determinantal form for the area will be evident. The transformation equations are as easily deduced by the direct application of Theorems I and II. The application of these three theorems to three dimensions is equally successful.

I do not at all mean to imply that these general proofs should be used to the exclusion of the time honored method of taking a figure in the first quadrant and deducing the result geometrically. These geometrical exercises may well be relegated to the problem lists. But the general method is the simpler, and the knowledge of the vector requisite for its use is not beyond the college freshman. For many students the vector idea can be introduced none too early. Anyone contemplating a new text on analytical geometry should certainly weigh these possibilities.

RECENT PUBLICATIONS.

REVIEWS.

Mathematical Logic.

A Survey of Symbolic Logic. By C. I. Lewis. Berkeley, University of California Press. 1918. Royal 8vo. 6 + 409 pp. Price $4.00.

Molière's M. Jourdain was very much surprised when told that he had been using prose all his life. Equally astonished are many present-day mathematicians when informed that they have been using 'logical prose'—propositional functions, the Zermelo axiom, and the like—for a correspondingly long period.

What is this logical prose of which the mathematical and the logical world at large have been, till quite recently, so blissfully ignorant? It is the principles of modern deductive logic, known also as symbolic or mathematical logic. Though Professor Lewis prefers the term *symbolic,* Russell and his school seem to have established almost irrevocably the name *mathematical* logic. And Professor Lewis's book is a survey of the history of the various stages in the discovery of the principles of deductive logic.

What are these principles? Everyone has heard of the famous 'Laws of Thought'—the Laws of Identity, Contradiction, and Excluded Middle. Assuming that these laws, considered as principles of *logic* (not of thought), are necessary, are they also sufficient? Obviously not; for the principle of the Syllogism is just as necessary to logical procedure as are these laws. Are the four principles sufficient? How shall we decide? We can study the problem empirically.